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REGRESSION ANALYSIS OF A PENDULUM'S OSCILLATION PERIOD

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ABSTRACT

We applied the regression analysis techniques to the data obtained from a simple pendulum experiment. Our data consist of a series of combinations of different variables, weight, length, angle, and period of oscillation. This data is investigated the least-squares linear regression analysis. The estimated model satisfies all the conditions of a best regression model. We found that there is statistically significant positive relationship between time and the other variables length and initial angle.

Keywords: Pendulum, Regression, Estimation, Residual.
JEL: C13, C21, C24, C26.

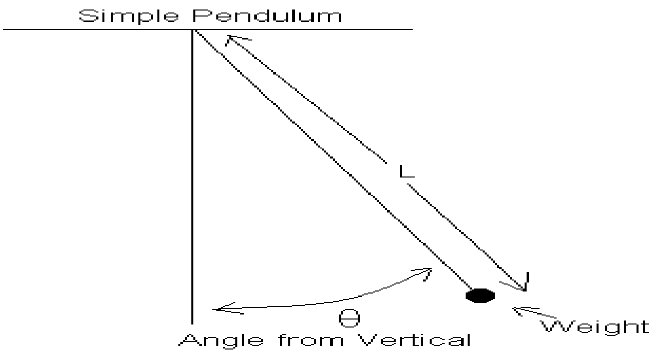
1. INTRODUCTION

It is well known that the oscillation period of a pendulum is given by the formula $T \approx 2\pi\sqrt{\frac{L}{g}}$ or in its full form $T \approx 2\pi\sqrt{\frac{L}{g}}\ (1 + \frac{1}{16}\theta_0^2 + \frac{11}{3072}\theta_0^4 \ldots)$ where L is the length of the pendulum, g represents the gravitational constant, approximately 9.8 for earth, and θ_0 is the initial angle the of release for the pendulum Figure(1). The aim of the study is to show that this relation can be proved using statistical modeling showing that science and statistics go hand in hand. This report has the following two main aims: To prove that the weight does not affect the swing time of a pendulum, and find an accurate linear model for the approximation of swing time, and in turn compare it to the full formula to test its accuracy.

Symon (1960) has derived an approximation formula for the period of a simple pendulum suggested earlier by Kidd and Fogg. It is well known that the equation of motion of a simple pendulum is nonlinear and consequently that the period T depends on the amplitude θ_0. The exact expression is:

$$T = 4\sqrt{\frac{L}{g}}K(k)$$

where L is the length of the pendulum, g is the acceleration due to gravity, and K(k) is the elliptic integral of the first kind. The modulus k is equal to sin ($\theta_0/2$).
Figure(1): Simple pendulum.



2. METHOD AND MATERIALS

2.1 Variables of Interest

In this study we considered four different variables, 3 predictors, and one response. These variables are: Length (categorical) - five different pendulum lengths are measured and recorded in millimeters. Weight (categorical) - three different weights (a,b,c). Angle of release (categorical)- four different angles of release are measured and recorded in degrees from the center line (the pendulum's equilibrium position). Time (continuous)- The time of one period of oscillation is recorded in seconds. A period of oscillation is defined as the time taken for the pendulum to complete one cycle finishing in its starting position.

Table 1: Full list of the predictor variables and their coinciding categorical value

Length		Angle		Weight	
750 mm	L1	15°	A1	Lightest	W1
600 mm	L2	30°	A2	Middle	W2
500 mm	L3	45°	A3	Heaviest	W3
350 mm	L4	60°	A4		
250 mm	L5				

2.2 Data Collection

The experiment is based on complete factorial design. This involve an equal number of repetitions of each possible combination of weight, angle and weight being measured. The angles measured are 15, 30, 45 and 60 degrees while the lengths are 750, 600, 500, 350 and 250 mm and three different weights are measured. All the measurements are filmed, and by using a computer to slow down the pendulum motion, we managed to obtain more accurate readings. A full list of the data is recorded see Appendix.

2.3 Regression Analysis

In statistics, regression analysis is a statistical process for estimating the relationships among variables. It includes many techniques for modeling and analyzing several variables, when the focus is on the relationship between a dependent variable and one or more independent variables. More specifically, regression analysis helps one understand how the typical value of the dependent variable (or 'criterion variable') changes when any one of the independent variables is varied, while the other independent variables are held fixed. Most commonly, regression analysis estimates the conditional expectation of the dependent variable given the independent variables that is, the average value of the dependent variable when the independent variables are fixed. Less commonly, the focus is on a quintile, or other location parameter of the conditional distribution of the dependent variable given the independent variables. In all cases, the estimation target is a function of the independent variables called the regression function. In regression analysis, it is also of interest to characterize the variation of the dependent variable around the regression function which can be described by a probability distribution.

Regression analysis is widely used for prediction and forecasting, where its use has substantial overlap with the field of machine learning. Regression analysis is also used to understand which among the independent variables are related to the dependent variable, and to explore the forms of these relationships. In restricted circumstances, regression analysis can be used to infer causal relationships between the independent and dependent variables. However this can lead to illusions or false relationships, so caution is advisable, (Armstrong, 2012).

Many techniques for carrying out regression analysis have been developed. Familiar methods such as linear regression and ordinary least squares regression are parametric, in that the regression function is defined in terms of a finite number of unknown parameters that are estimated from the data. Nonparametric regression refers to techniques that allow the regression function to lie in a specified set of functions, which may be infinite-dimensional.

2.4 Nonlinear Regression Analysis

In statistics, nonlinear regression is a form of regression analysis in which observational data are modeled by a function which is a nonlinear combination of the model parameters and depends on one or more independent variables. The data are fitted by a method of successive approximations. The data consist of error-free independent variables (explanatory variables), x, and their associated observed dependent variables (response variables), y. Each y is modeled as a random variable with a mean given by a nonlinear function f(x,β). Systematic error may be present but its treatment is outside the scope of regression analysis. If the independent variables are not error-free, this is an errors-in-variables model, also outside this scope. For example, the Michaelis–Menten model for enzyme kinetics

$$v = \frac{V_{\max}\,[\mathrm{S}]}{K_m + [\mathrm{S}]}$$ can be written as $$f(x, \boldsymbol{\beta}) = \frac{\beta_1 x}{\beta_2 + x}$$

where β_1 is the parameter $V_{\max}$, β_2 is the parameter K_m and [S] is the independent variable, x. This function is nonlinear because it cannot be expressed as alinear combination of the two βs.

Other examples of nonlinear functions include exponential functions, logarithmic functions, trigonometric functions, power functions, Gaussian function, and Lorenz curves. Some functions, such as the exponential or logarithmic functions, can be transformed so that they are linear. When so transformed, standard linear regression can be performed but must be applied with caution. See Linearization, below, for more details.

In general, there is no closed-form expression for the best-fitting parameters, as there is in linear regression. Usually numerical optimization algorithms are applied to determine the best-fitting parameters. Again in contrast to linear regression, there may be many local minima of the function to be optimized and even the global minimum may produce a biased estimate. In practice, estimated values of the parameters are used, in conjunction with the optimization algorithm, to attempt to find the global minimum of a sum of squares. For details concerning nonlinear data modeling see least squares and non-linear least squares.

2.5 Transformation

Some nonlinear regression problems can be moved to a linear domain by a suitable transformation of the model formulation. For example, consider the nonlinear regression problem $y = \alpha e^{bx} U$ with parameters a and b and with multiplicative error term U. If we take the logarithm of both sides, this becomes $Ln(y) = Ln(\alpha) + bx + U$ where u = Ln(U), suggesting estimation of the unknown parameters by a linear regression of Ln(y) on x, a computation that does not require iterative optimization. However, use of a nonlinear transformation requires caution. The influences of the data values will change, as will the error structure of the model and the interpretation of any inferential results. These may not be desired effects. On the other hand, depending on what the largest source of error is, a nonlinear transformation may distribute your errors in a normal fashion, so the choice to perform a nonlinear transformation must be informed by modeling considerations.

3. LINEAR REGRESSION ANALYSIS

So far we have considered only one regressor X besides the constant in the regression equation. The relationships between variables usually include more than one regressor. For example, a demand equation for a product will usually include real price of that product in addition to real income as well as real price of a competitive product and the advertising expenditures on this product. In this cases:

$$y_i = \alpha + \beta_1 x_{1i} + \beta_2 x_{2i} + \cdots + \beta_k x_{ki} + u_i \quad i = 1.2.3 \ldots\ldots, n$$

where $y_i$denotes the i-th observation on the dependent variable x_{ki} denotes the i-th observation on the independent variable x_k for k = 1, . . . , K. α is the intercept and β1, β2, .. , β_K are the (K– 1) slope coefficients. The u_i denotes the i-th random errors.

3.1 Assumption

The assumptions underlying linear regression are:

- The relationship between the independent X variable(s) and the dependent Y variable is linear.
- The residuals follow a normal probability distribution. Thus, the difference between each actual value of Y and the estimated value of Y follow a normal distribution.
- The variance around the regression line (residual) is the same for all the values of the independent variable. The term for this constant variance of the errors is homoscedasticity. If the error variance is not constant, then hetroscedicity is present.
- The independent variables should not be correlated with one another. When independent variables are correlated, the term for this condition is Multicollinearity.

3.2 Estimation

The linear regression model in equation(1) the random errors assuming $\mathrm{E}(U_i) = 0$ and $\mathrm{Var}(U_i) = \sigma^2$. In the classical regression setting the error term is assumed to be normally distributed with a constant variance σ^2. The regression coefficients are estimated using the least squares principle. It should be noted that it is not necessary to assume that the regression error term follows the normal distribution in order to find the least squares estimation of the regression coefficients. It is rather easy to show that under the assumption of normality of the error term, the least squares estimation of the regression coefficients are exactly the same as the maximum likelihood estimations (MLE) of the regression coefficients. The multiple linear model can also be expressed in the matrix format which is defined by the following vectors and matrices.

$$\mathrm{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} \quad \mathrm{X} = \begin{bmatrix} 1 & X_{11} & X_{12} & \cdots & X_{1,p-1} \\ 1 & X_{21} & X_{22} & \cdots & X_{2,p-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & X_{n1} & X_{n2} & \cdots & X_{n,p-1} \end{bmatrix} \quad \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_{p-1} \end{bmatrix} \quad \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

Except for the fact that the dimensions of the vectors and matrices are different, they represent the same form as was used in simple linear regression.

$$Y = X\beta + \varepsilon$$

where : Y is a (n×1) vector of responses, β is a (p×1) vector of parameters, X is a (n×p) matrix of constants, ε is a (n×1) vector of independent random variables with such that

$$\mathrm{E}\{\varepsilon\} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \text{and} \quad \sigma^2\{\varepsilon\} = \begin{bmatrix} \sigma^2 & 0 & \cdots & 0 \\ 0 & \sigma^2 & \cdots & \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \sigma^2 \end{bmatrix}$$

3. 3 Residual Analysis

Residual analysis is essential for assessing the assumptions of the linear model and is helpful for identifying outliers. Each residual is the difference between an observed value and the value predicted by the regression equation. The residual of the linear regression model y = Xβ + ε is defined as the difference between observed response variable y and the fitted value y ˆ , i.e., e = y - ˆ y . The regression error term ε is unobservable and the residual is observable. Residual is an important measurement of how close the calculated response from the fitted regression model to the observed response. The purposes of the residual analysis are to detect model mis-specification and to verify model assumptions. Residuals can be used to estimate the error term in regression model, and the empirical distribution of residuals can be utilized to check the normality assumption of the error term (QQ plot), equal variance assumption, model over-fitting, model under-fitting, and outlier detection. Overall, residual analysis is useful for assessing a regression model.

3.4 Characteristics of Best model

Independent variables is very high interpreted of variation on dependent variable. Tested by the value of the coefficient of determination R^2 greater than (0.60%). Most Independent variables should be individual significant to explain dependent variable as expressed in the format below:

$T = \frac{\hat{\beta}}{S_{\hat{\beta}}}$ Reject $H_0 : \beta_i = 0$ if p-value less than α, (example, α =0.05)

Independent variables should be jointly significant to explain dependent variable. Testing Global Usefulness of the Model l: The Analysis of Variance F-Test

$H_0 : \beta_1 = \beta_2 = \cdots = \beta_k = 0$ (All model terms are unimportant for predicting y.

H_1: At least one $\beta_i \neq 0$ (At least one model term is useful for predicting y)

Test statistic $F = \frac{MSR}{MSE}$ Reject H_0 if P-value is less than α. There is no Multicolinearity correlation between the independent variables.

Tested by correlations coefficient matrix (R) between the independent variables. Finally, there is no hetroscedicity in the residual. There are several tests used to detect problem of hetroscedicity, White Test, ARCH , Breusch-Pagan-Godfrey Test. Residual are normally distributed. The probability value of Jarque-beta for the Normality Test is greater than (0.05).

4. RESULTS AND DISCUSSION

The overall model used in this study consists of a number variables expressed as follows:

$$T = 4\sqrt{\frac{L}{g}}K(k)$$

$$LnT = \text{Ln}4 + \frac{1}{2}\text{Ln}L - \frac{1}{2}\text{Ln}g + LnK(k)$$

$$LnT = (\text{Ln}4 - \frac{1}{2}\text{Ln}g) + \frac{1}{2}\text{Ln}L + LnK(k)$$

The Linear regression Model formula can be written as

$$LnT = \beta_0 + \beta_1 \text{Ln}L + \beta_2 LnK + \mu_i$$

The Estimation of Linear regression Model can be written as

$$L\hat{n}T = \hat{\beta}_0 + \hat{\beta}\,\text{Ln}L + \hat{\beta}_2\,LnK$$

The regression output is shown in Table(1) labeled coefficients contains values for the β's, t-statistics, and p-values. The values for the β's, are less than (0.05) indicate that there is a statistically significant relationship between dependent variable Time and independent (Angle, Length, Weight). and significant effect of these variables on the dependent variable.

Table 2: Estimation of the pendulum Oscillation Period model

Dependent Variable: Ln Time				
Method: Least Squares				
Date: 04/26/15 Time: 21:33				
Sample (adjusted): 1 119				
Included observations: 119 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-2.827920	0.078428	-36.05775	0.0000
Ln length	0.497581	0.011821	42.09213	0.0000
Ln angle	0.030457	0.008894	3.424379	0.0009
R-squared	0.939100	Mean dependent var		0.323169
Adjusted R-squared	0.938050	S.D. dependent var		0.201769
S.E. of regression	0.050220	Akaike info criterion		-3.119925
Sum squared resid	0.292556	Schwarz criterion		-3.049863
Log likelihood	188.6355	Hannan-Quinn criter.		-3.091475
F-statistic	894.3805	Durbin-Watson stat		1.914085
Prob(F-statistic)	0.000000			

Source : Data analysis using (EVIWES) Software

Table 3 shows that:

a) The estimation model can be written as:

$$Ln\,\hat{T}ime = -2.827920 + 04975\ \text{Ln}(Lenght) + 0.03045Ln(Angle)$$

b) The value of the coefficient of determination R^2 of the pendulum Oscillation Period model is (0.93%) and this indicates That 93 % of the changes that occur in the dependent variable resulting from changes that occur in the independent variables and 7% resulting from the change in the random variables.

c) The β's, t-statistics, and p-values. The p-values for the β's, are less than (0.05) indicate that there is a statistically significant relationship between dependent variable Time and independent (Angle, Length, and constant.

d) The problem is discovered autocorrelation by statistical Durbin - Watson , If the value of the Durbin - Watson close (2) or equal .this indicates that the absence of the equation problem autocorrelation. If the value is smaller than (1.5), this indicates that the presence of a positive self- correlation and if the value is greater than (2.5) indicates that the presence of a self- correlation minus. Table (2) above show that the statistical value of Durbin–Watson is (1.955) indicating that the is no Auto correlation problem.

The model suitability tests are statistically fit for purpose and verified to find correlations between the independent variables.

Table 3: correlations matrix between the independent variables

	Angle	Length	Weight
Angle	1	-0.01533649	0.01400221
Length	-0.015336490	1	0.01682177
Weight	0.01400221	0.01682177	1

Source : Data analysis using (EVIWES) Software

Table 3 shows that estimation equation of the pendulum Oscillation Period model without a problem multicolinearity between the independent variables. There are several tests used to detect the hetroscedicity problem, including Breusch- Godfrey

Table 4: Breusch-Pagan-Godfrey Test

F-statistic	1.884854	Prob. F(3,115)	0.1360
Obs*R-squared	5.577019	Prob. Chi-Square(3)	0.1341
Scaled explained SS	5.552856	Prob. Chi-Square(3)	0.1355
F-statistic	1.884854	Prob. F(3,115)	0.1360

Source : Data analysis using (EVIWES) Software

Table 4 shows that the probability value Obs*R-squared for the hetroscedicity Test: Breusch-Pagan-Godfrey is greater than the significance level (0.05) , indicating that the residual of the pendulum Oscillation Period model suffers from no problem of hetroscedicity.

5. CONCLUSION

In this paper we estimated the pendulum Oscillation Period by using a linear regression model. The model was than evaluated and examined by some statistical criteria. We find that the value of the coefficient of determination model is (0.98), this indicates the goodness of the model. We also find that there are statistically significant positive relationship between time and the dependent variables, length and initial angle. The estimated model satisfies all the conditions of a best regression model, no serial correlation, no hetroscedicity and residual are normally distributed.

In this study we have managed to provide a sufficient statistical evidence that weight does not affect the swing time of a pendulum, while length and angle of release do. From this we are also able to create a new linear equation for the swing time, which incorporated both the angle and the length.

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Appendix 1: Observation Chart Of Pendulum Swings

No	Time	Angle	Length	Weight	No	Time	Angle	Length	Weight
1	1.5	45	500	1	61	1.9	15	750	3
2	1.5	45	500	1	62	1.8	15	750	3
3	1.5	60	500	1	63	1.7	30	750	3

4	1.6	60	500	1	64	1.7	30	750	3
5	1.1	15	350	1	65	1.6	45	750	3
6	1.1	15	350	1	66	1.8	45	750	3
7	1.2	30	350	1	67	1.7	60	750	3
8	1.1	30	350	1	68	1.9	60	750	3
9	1.3	45	350	1	69	1.6	15	600	3
10	1.2	45	350	1	70	1.5	15	600	3
11	1.2	60	350	1	71	1.6	30	600	3
12	1.2	60	350	1	72	1.5	30	600	3
13	1	15	250	1	73	1.6	45	600	3
14	1	15	250	1	74	1.5	45	600	3
15	1	30	250	1	75	1.5	60	600	3
16	1	30	250	1	76	1.6	60	600	3
17	1.1	45	250	1	77	1.5	15	500	3
18	1.2	45	250	1	78	1.4	15	500	3
19	1	60	250	1	79	1.5	30	500	3
20	1.1	60	250	1	80	1.3	30	500	3
21	1.8	15	750	2	81	1.5	45	500	3
22	1.9	15	750	2	82	1.4	45	500	3
23	1.8	30	750	2	83	1.4	60	500	3
24	1.8	30	750	2	84	1.5	60	500	3
25	1.8	45	750	2	85	1.1	15	350	3
26	1.9	45	750	2	86	1.2	15	350	3
27	1.8	60	750	2	87	1.2	30	350	3
28	1.8	60	750	2	88	1.1	30	350	3
29	1.5	15	600	2	89	1.2	45	350	3
30	1.5	15	600	2	90	1.2	45	350	3
31	1.6	30	600	2	91	1.3	60	350	3
32	1.5	30	600	2	92	1.2	60	350	3
33	1.5	45	600	2	93	1.1	15	250	3
34	1.6	45	600	2	94	1	15	250	3
35	1.6	60	600	2	95	1	30	250	3
36	1.6	60	600	2	96	1	30	250	3
37	1.5	15	500	2	97	1	45	250	3
38	1.6	15	500	2	98	1.1	45	250	3
39	1.4	30	500	2	99	1.1	60	250	3
40	1.5	30	500	2	100	1.1	60	250	3
41	1.5	45	500	2	101	1.8	15	750	1
42	1.4	45	500	2	102	1.7	15	750	1
43	1.5	60	500	2	103	1.8	30	750	1
44	1.5	60	500	2	104	1.9	30	750	1
45	1.1	15	350	2	105	2	45	750	1
46	1.2	15	350	2	106	1.9	45	750	1
47	1.1	30	350	2	107	1.7	60	750	1
48	1.2	30	350	2	108	1.9	60	750	1
49	1.3	45	350	2	109	1.5	15	600	1
50	1.2	45	350	2	110	1.5	15	600	1

51	1.2	60	350	2	111	1.7	30	600	1
52	1.2	60	350	2	112	1.5	30	600	1
53	1	15	250	2	113	1.6	45	600	1
54	1	15	250	2	114	1.6	45	600	1
55	1.1	30	250	2	115	1.6	60	600	1
56	1.1	30	250	2	116	1.6	60	600	1
57	1	45	250	2	117	1.4	15	500	1
58	1	45	250	2	118	1.4	15	500	1
59	1.1	60	250	2	119	1.5	30	500	1
60	1	60	250	2	120	1.5	30	500	1

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