PUBLIC EXPENDITURES SHOCKS IN A REAL BUSINESS CYCLE MODEL: IMPLICATIONS FOR THE ECONOMY OF TUNISIA

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ABSTRACT

The effects of the increase in Tunisian government expenditure on the private sectors, is investigated in a Dynamic Stochastic General Equilibrium Model. We establish a distinction between the consumption and the investment expenditure components of government purchases because these shocks are expected to generate different macroeconomic effects. There are two main findings: (i) the economy’s response to an increase in government expenditure depends on how it is financed. Distortionary tax finance may lead to a decline in output, consumption, and investment. In contrast, lump-same tax finance increases output and employment. (ii) Government investment differs from government consumption in that it increases output similarly to a positive technology shock. These findings lead us to conclude that the fiscal stimulus package through additional temporary government spending that has been implemented this year may help to achieve social goals while stimulating economic activity and employment.

Keywords: DSGE Model, Fiscal Shock, Business Cycle, Public Capital

\textit{JEL Classifications:} E02, E6, E44, H5, H54, H72.

1. INTRODUCTION

Fiscal policy has undergone significant changes in empirical macroeconomics in the last decade. Simulation of the real effects of government spending is increasingly based on dynamic stochastic general equilibrium models (DSGE). This new trend overshadowed the old approach that was based on the Keynesian or so-called synthesis macroeconomic framework of macroeconomic supply and demand. As is well know today, this old framework was strongly criticized for lack of internal consistency by R. Lucas in his famous “Lucas Critique” (1976). According to the latter, the major weakness of the traditional Keynesian model is that it assumes invariance of the reaction coefficients of real variables with regard the exogenous fiscal or monetary policy instruments. Lucas forcefully argues that parameters of the aggregate functions, which are supposed to represent the behaviour of the economic agents, do not constitute fundamental parameters of the economy, and that disturbances will affect these parameters and thus the slopes of the supply and demand curves.
The new approach of DSGE has addressed Lucas’ challenge by focusing on the microfoundations of the behavioural relations which are the building blocks of any useful macroeconomic model. These relations are anchored in the inter-temporal optimization framework. This new empirical macroeconomics has been applied mainly for developed economies while developing economies have received little attention so far in these applications, in spite of its relevance to tackle real policy issues such as fiscal policy. Countries such as Tunisia (Egypt and perhaps other countries to come) are currently faced with sharp shocks such as temporary expansionary fiscal policy needed to respond to the new aspirations of their citizens. Additional budgets have been put in place in order to accommodate these new expectations.

The objective of this paper is to build a DSGE for Tunisia and to use it for the simulation of such policy shocks. We would like to assess whether additional temporary government spending is likely to help achieve merely social goals or to result in real effects on economic activity and employment.

We are aware that the Real Business Approach is controversial, allegedly assuming too much rationality on the part of economic agents. Its findings on the effects of shocks on the private sector, on investment, consumption, employment, economic activity and wages are debatable. However, its merits of internal consistency and relevance for macroeconomic shocks should not be underrated. In any case, the findings should be compared to those that may be obtained using alternative macroeconomic models. With the limitations of this exercise in mind, our objective is to build and calibrate an adapted DSGE and use it to simulate temporary fiscal shocks such as the ongoing ones in Tunisia.

This paper proceeds as follows: section 1 discusses fiscal policy in Tunisia after the revolution. Section 2 reviews the literature on DSGE. Section 3 provides some stylized facts about the business cycle for Tunisian economy. The theoretical model is presented in section 4. Model resolution is carried out in section 5 and section 6 simulates the impact of fiscal shocks on the dynamic of the economy.

1.1 FISCAL POLICY IN TUNISIA AFTER THE REVOLUTION

The revolution of January 14, 2011, has had severe short-term macroeconomic consequences. In addition to property damage (estimated at 4% of GDP), Tunisia has been faced, on the domestic front, by a rise in insecurity and social tensions, and a collapse in tourism receipts (-46%) as well as a sharp decline in FDI (-17.8%). External finance to support funding of projects and Tunisian companies has been discouraged by higher country risk. As a result, unemployment has risen in a few months from 14% to close to 18% of the active population. In order to address these severe problems the provisional Government has drafted a medium-term plan, labelled “The Economic and Social Development plan or Jasmine Plan, the objective of which is to set sound conditions for future prosperity and consolidation of the nascent democracy. The Jasmine Plan has been articulated along a three-pronged approach: (i) Post-revolution crisis management and political reform, (ii) carrying out the transition towards democracy, and (iii) setting forth the conditions for sustained social and economic development in the medium term.

During the post-revolution crisis management stage, the Economic and Social Development plan recommended a short-term increase in public expending by 11% in order to respond to social aspirations but also to stimulate economic activity. In this context, the short term economic and social emergency measures include job-support programs, enhanced financial support for social and regional development, assistance to those enterprises that were adversely affected by social disturbances, as well as various fiscal incentives aimed at reactivating the economy.
Table 1. Public Finance in Tunisia (percentage of GDP)

<table>
<thead>
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<tr>
<td>Total revenue and</td>
<td>23.3</td>
<td>21.9</td>
<td>24.2</td>
<td>23.1</td>
<td>23.1</td>
<td>23.3</td>
<td>23.1</td>
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<tr>
<td>Grant</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Tax revenue</td>
<td>19.5</td>
<td>19.1</td>
<td>20.5</td>
<td>19.9</td>
<td>20.1</td>
<td>20.2</td>
<td>20.1</td>
</tr>
<tr>
<td>Grants</td>
<td>0.4</td>
<td>0.1</td>
<td>0.3</td>
<td>0.3</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Total expenditure</td>
<td>25.6</td>
<td>24.5</td>
<td>24.8</td>
<td>25.8</td>
<td>25.7</td>
<td>28.5</td>
<td>27.9</td>
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<tr>
<td>and net lending</td>
<td></td>
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<td></td>
<td></td>
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<td></td>
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<tr>
<td>Current expenditure</td>
<td>18.0</td>
<td>18.4</td>
<td>19.2</td>
<td>18.1</td>
<td>18.4</td>
<td>21.0</td>
<td>20.6</td>
</tr>
<tr>
<td>Wage and salaries</td>
<td>11.2</td>
<td>10.6</td>
<td>10.4</td>
<td>10.7</td>
<td>10.7</td>
<td>11.4</td>
<td>12.0</td>
</tr>
<tr>
<td>Good and services</td>
<td>1.5</td>
<td>1.6</td>
<td>1.6</td>
<td>1.7</td>
<td>1.9</td>
<td>2.3</td>
<td>2.3</td>
</tr>
<tr>
<td>Interest</td>
<td>2.8</td>
<td>2.4</td>
<td>2.1</td>
<td>2.1</td>
<td>1.9</td>
<td>1.9</td>
<td>1.7</td>
</tr>
<tr>
<td>Capital expenditure</td>
<td>7.3</td>
<td>5.4</td>
<td>5.8</td>
<td>6.6</td>
<td>7.2</td>
<td>7.3</td>
<td>7.1</td>
</tr>
<tr>
<td>Primary balance</td>
<td>0.5</td>
<td>-0.2</td>
<td>1.4</td>
<td>-0.7</td>
<td>-0.7</td>
<td>-3.3</td>
<td>-3.0</td>
</tr>
<tr>
<td>Overall balance</td>
<td>-2.3</td>
<td>-2.6</td>
<td>-0.7</td>
<td>-2.7</td>
<td>-2.6</td>
<td>-5.2</td>
<td>-4.8</td>
</tr>
</tbody>
</table>

Source: ADB African Economic Outlook 2011

As illustrated in table 1, the budget deficit was kept below 3% in 2009 and 2010. To achieve this budget deficit target, the strategy was based on the control of current spending and subsidies combined with relatively high public investment. But in the aftermath of the revolution, the Government has sharply increased its current spending in 2011 and is expected to maintain almost the same spending in terms of GDP in 2012. In addition to creating a large number of jobs in the public sector, the government continued the help that was provided to exporting firms in 2010 with funding to encourage job creation and boost their income, and also increased subsidies for fuel, food staples and transport. Prices of these subsidised items were expected to rise gradually as subsidies were reduced, but the revolution has changed all this and the TND 1.5 billion in subsidies in 2010 has risen to 2 billion in the 2011 budget.

Investment spending already increased in 2010 in order to address the adverse effects of the international financial crisis on the Tunisian economy. A large number of public infrastructure projects are included in the Jasmine Plan, although at a stabilised ratio in terms of GDP. As a result of the rise in public spending, the budget deficit is expected to rise sharply, from 2.6% in 2010 to 5.2% of GDP in 2011 and 4.8% in 2012 (Table 1).

2. THEORETICAL AND EMPIRICAL LITERATURE REVIEW

The assessment of the effects of government expenditures on the behavior of the private sector over the business cycle is crucial for understanding the effectiveness of stimulus packages in recessions, and for distinguishing between competing predictions of alternatives macroeconomic models. Even though the effects of fiscal policy are of central importance in macroeconomics, there is no consensus on its impact and transmission channels. Both theoretical and empirical studies generally find a positive response of output and hours worked to a positive shock to government purchases while the sign of the responses of variables such as consumption, wages and investment is still a matter of debate RBC theory, which is the main theoretical framework, generally predicts a positive response of investment and negative response of consumption and wages to a positive shock to government purchases. In contrast, IS-LM theory predicts that consumption should rise and thus amplify the expansionary effects of government spending on output.

RBC models with fiscal policy shocks are based on the neoclassical growth model of capital accumulation. The model incorporates forward looking agents and rational expectations. The modeling strategy is as follows: all agents in the model follow optimal plans. Households seek to maximize lifetime expected utility subject to their budget constraint. Firms maximize profits subject to the available technology, and the government has to meet its budget constraint. The solution of a constrained inter-temporal optimization problem is represented by the time-paths of aggregate consumption, employment, investment, and output. The source of disturbances in the model environment is, in addition to a sequence of technology shocks, a sequence of stochastic fiscal measures. An exogenous state vector of disturbances is driven by a
stationary AR (1) process with technology and fiscal variables. The government and private sector are modeled as being subject to a budget constraint requiring income to equal expenditures dynamically. There are a lot of typical questions related to the construction of RBC models when government sector is incorporated:

- How do temporary and/or permanent changes in government expenditures influence the dynamic of macroeconomic aggregates if the changes are financed from lump-sum taxes?
- How do changes in distortional taxes influence the dynamic of macroeconomic aggregates if the tax benefits are refunded to private agents in the form of lump-sum transfers?
- How do temporary and/or permanent changes in government expenditures influence the dynamic of macroeconomic aggregates if the changes are financed from distortional taxes?

The early analysis can be found in the neoclassical studies of the real macroeconomic impact of changes in government purchases (Hall [1980], Barro [1981]) or taxation (Auerbach et al. [1983]). The findings of Hall [1980] and Barro [1981] indicated that the effects of transitory increases in government consumption were larger than the similar effects of persistent increases in a standard neoclassical growth model.

Christiano and Eichenbaum [1992] incorporate stochastic changes in public expenditures to improve the capability of basic RBC model to stimulate an observed low correlation between labor productivity and employment. Government expenditures were nonproductive purchases financed from lump-sum taxes; the influence of such purchases impacted only the utility functions of householders. The substitution’s coefficient between private and public consumption was set equal to zero meaning that an increase in public consumption was formally equal to the net outflow of resources from an economy. Hence, individuals react to this shock as if they lost a part of their wealth by increasing the number of hours worked.

Aiyagari et al. [1992] used an RBC model with government purchases, lump-sum taxes and utility function with additively separated preferences between private and public consumption. The authors found the quantitative effects of a persistent increase in government expenditures to be always larger than those of a temporary increase. The authors provided evidence that if government expenditures increase, the interest rate always increases, no matter if an increase of government expenditure was temporary or permanent and, that if changes in the level of public expenditures were sufficiently persistent, the output effects of the change exceeded unity and there was a mechanism similar to a Keynesian multiplier working in the conventional neoclassical model environment.

Baxter and King [1993] constructed an RBC model to investigate the macroeconomic effects of permanent changes in government purchases and how these effects differed from those of transitory changes. They also examined how the mode of financing government spending affects its macroeconomic impacts. In addition, they examined the quantitative impact of public investment in both the long and short run. Their main findings are as follows: first, permanent changes in government purchases have important effects on macroeconomic activity when these are financed by lump sum taxation. Second, permanent changes in government purchases are associated with larger effects than transitory changes in government purchases, contrary to the findings of Hall [1980] and Barro [1981]. Third, the financing decision is quantitatively much more important than the direct resource cost of government purchases.

McGrattan [1994] constructed a RBC model with government spending and distortional taxation to explore households’ responses to tax rate fluctuations, and substituting between taxable and non-taxable activities. The model incorporated the conventional assumptions of non-productivity of government purchases and a zero value for the substitution coefficient associated with the utility function containing government and private consumption. The author
investigated the simulation of two versions of the RBC model: the first is represented by a model with variable tax rates and government consumption and the second, by a model with constant tax rates and government consumption. The results indicate that a model with variable fiscal parameters appear more quantitatively capable to explain the historical data than a model with constant fiscal parameters.

The need for empirical evidence to clarify these issues has stimulated a large body of new research which can be loosely grouped in three categories. The first focused on the macroeconomic effects of fiscal consolidation. The second line of research analyzed the effectiveness of fiscal policy through the stabilization capability of fiscal policy variables, i.e. to what extent the tax and transfer system provides insurance against idiosyncratic shocks and how well it stabilizes macroeconomic fluctuations. Finally, the dynamic effect of discretionary fiscal policy on macroeconomic variables was recently revived within the framework of vector autoregression (VAR).

The empirical analysis of the effects of fiscal policy is mainly conducted using vector-autoregression (VAR) even though it has confronted great difficulties in identifying fiscal shocks\(^1\). The first approach, commonly referred to as narrative, is based on the argument that increases in military spending are good indicators of unanticipated policy shifts, which motivates the use of dummy variables to identify unanticipated increases in military purchases in the U.S. economy. The methodology consists of constructing a dummy variable that takes the value of one at quarters when large military build-ups took place in the U.S, in order to identify episodes of discretionary fiscal policy. Shapiro [1998] uses this methodology in univariate setting. Edelberg, Eichenbaum and Fisher [1999] extended this methodology to multivariate context, and Burnside, Eichenbaum and Fisher [2004], as well as Eichenbaum and Fisher [2005] made some modifications. Despite slight methodological differences, all these studies generally reached the same conclusions: in response to a discretionary positive government spending shock, output increases, consumption and wages decline, non-residential investment rises, while residential investment falls. Auerbach [2010] argues that this approach focuses on the effect of specific types of shocks and cannot be generalized to evaluate broader policy effects such as short-run recessionary periods.

The second strand employs an identification strategy using structural autoregression methodology in order to trace the impulse responses of the macroeconomic variable of interest. Fatas and Mihov [2001] found that there is a strong, positive, and persistent impact of fiscal policy expansions on economic activity. Blanchard and Perotti [2002] achieved identification by relying on institutional information about tax collection, constructing the automatic stabilizers and, by implication, identifying discretionary fiscal policy shocks. The Blanchard-Perotti approach yields a positive effect of a government spending shocks on output and consumption and negative effect on investment. While these findings are perfectly reasonable in a Keynesian framework, they are difficult to reconcile with RBC literature.

3. BUSINESS CYCLE STYLIZED FACTS OF TUNISIAN ECONOMY

We start our study by identifying stylized facts about the Tunisian economy in terms of the economic fluctuations. Following the approach widely used in developed countries, we use the HP filter after taking natural logarithms to decompose the Tunisian aggregate data into a trend component and a cyclical component.

Some selected statistics summarizing the business cycles moments are provided in Table1. The following will generalize the stylized facts of the business cycle in the Tunisian economy by focusing on three statistical moments: the volatility and persistence of the detrended variables and the co-movements between macroeconomic aggregates.

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\(^1\) To distinguish between the role of automatic stabilizers and the effects of discretionary fiscal policy
Table 2: Stylized Facts of Business Cycle for the Tunisian Economy

<table>
<thead>
<tr>
<th></th>
<th>Standard deviation</th>
<th>Relative standard deviation</th>
<th>First-order autocorrelation</th>
<th>Contemporaneous correlation with Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>3.06</td>
<td>1.00</td>
<td>0.51</td>
<td>1.00</td>
</tr>
<tr>
<td>Private consumption</td>
<td>1.93</td>
<td>0.63</td>
<td>0.34</td>
<td>0.79</td>
</tr>
<tr>
<td>Private investment</td>
<td>11.45</td>
<td>3.74</td>
<td>0.60</td>
<td>0.76</td>
</tr>
<tr>
<td>Public consumption</td>
<td>12.20</td>
<td>3.89</td>
<td>0.46</td>
<td>0.30</td>
</tr>
<tr>
<td>Public investment</td>
<td>12.50</td>
<td>4.08</td>
<td>0.61</td>
<td>0.72</td>
</tr>
</tbody>
</table>

Private consumption is slightly less volatile than output and private investment. This finding matches perfectly with consumption smoothing life cycle model and permanent income model. Private investment is almost three times more volatile than output. Public consumption and investment are more volatile than output. Their volatilities are about four times bigger than output respectively.

As the four columns of table 1 shows, all the detrended variables show some degree of first order persistence. For comparison, private consumption has the lowest persistence. The persistence of output, private investment and public investment are close and in the middle. As an important measure of such co-movement, the contemporaneous correlations of variables with output are calculated and reported in the last column of table 1. It seems that the spending components of output (private and public) consumption and investment are all procyclical, although the correlations of private consumption, private investment and public investment are higher than public consumption.

4. THE THEORETICAL MODEL

The theoretical model used in the study is a RBC model with government. The model economy is populated by a large number of households who maximize their life-time utilities, a large number of firms who are profit maximizers and a central government who finances its purchases by lump-sum taxes. Given the important role of the Tunisian government in the economy, we incorporate utility-yielding government consumption and productive public capital in the model. The production of firms is affected by technological progress which follows a first order autoregressive process. The government conducts its fiscal policy by varying the shares of its consumption and investment which also take the form of first-order autoregressive processes. For convenience, perfect competition rules the goods, capital and labour markets. All variables will be determined in a decentralized competitive equilibrium such that economic fluctuations are interpreted as deviations from this equilibrium resulting from exogenous shocks. The specification and derivation of the model are illustrated below. The RBC model with government, such as in Baxter and King [1993] is useful to answer four classic questions:

- Does an increase in government purchases lead to more than one-to-one increase in output?
- How do the effects of a permanent shock differ from those of a temporary shock?
- How does the introduction of distortionary taxes as opposed to lump-sum taxes alter the results?
- How is the analysis altered by introduction of public capital?
4.1. REPRESENTATIVE HOUSEHOLD

The economy consists of a large number of identical households. The representative household’s objective is to maximise its expected discounted flow of utility given preferences defined over effective consumption and leisure during each period $t = 0, 1, 2,...$

$$\max U = E_0 \sum_{i=0}^{\infty} \beta^i u(C_i, l_i)$$

$$u(C_i, l_i) = \frac{\left(\frac{C_i^{\alpha} l_i^{1-\alpha}}{\beta}\right)^{-\sigma} - 1}{1-\sigma}$$

Where $\sigma > 1$, is the coefficient of relative risk aversion, and $\alpha \in (0,1)$ is consumption’s share relative to leisure in instantaneous utility. With this utility form, the negative wealth effect of a government consumption shock reduces leisure, but raises the marginal utility of private consumption; if this effect is sufficiently strong, private consumption will increase after the shock. The household maximizes utility derived from consumption and leisure, knowing that each period it is constrained with one unit of time to be allocated between two activities:

$$1 = h_i + l_i$$

The idea of incorporating government consumption in the household’s utility functions follows Christiano and Eichenbaum [1992] and Baxter and King [1993]. Effective consumption $C_i$ is divided into private and public consumption. As in Barro [1981] we suppose that each unit of public consumption provides $\omega$ as many utilities as one unit of private consumption. Effective consumption $C_i$ is divided into private and public consumption. As in Barro [1981] we suppose that each unit of public consumption provides $\omega$ as many utils as one unit of private consumption.

$$C_i = C_i^p + \omega C_i^G$$

Where $C_i^p$ and $C_i^G$ denote private and public consumption, $\omega$ is the elasticity of substitution. This specification assumes that private and public consumption are perfectly substituted, but the $\omega$ coefficient shows how differently private and public consumption affect consumer’s satisfaction. Thus, following an increase of public expenditure by one unit, the representative household can reduce its private consumption by $\omega$ units while keeping the same level of welfare as initially. The household also has access to capital and labor markets. It received wage income, $h_i$, from supplying labour and gains capital income, $r_i K_i$, from holding capital. Since both capital and labour markets are perfectly competitive, $w_i$ and $r_i$ are real wage rate and market real rate of return to capital are equal to marginal products of labor and capital respectively. The household spends part of the net income on consumption and saves the remainder for investment. The intertemporal budget constraint of the household is given by:

$$C_i^p + I_i^p = (1-\tau_i) \frac{w_i}{\beta} L_i - T_i$$

$$C_i^p + I_i^p = (1-\tau_i) \frac{w_i}{\beta} L_i + r_i K_i - T_i$$
Where \( \tau^r \) and \( T_i \) are the rate of income taxation (capital income and labour income) and lump-sum taxes respectively. The change in private capital stock is given by:

\[
K^p_{t+1} = I^p_t + (1 - \delta^p)K^p_t 
\]

Since the labour and capital markets are competitive, each representative household takes market prices \((w_t, r_t)\) and policy instrument \(\left( \tau^r, T_i, C^G_t \right)\) as given. The household’s problem is then to maximise the lifetime utility by choosing \(\left\{ C^p_t, H_t, I^p_t \right\}_{t=0}^{\infty} \) subject to its budget constraint. By substituting the household investment into budget constraint, this optimisation problem is given by

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \left[ \left( \left( C^p_t + \alpha C^G_t \right)^{\vartheta} (1 - H_t)^{1-\vartheta} \right)^{1-\delta} \right] - 1
\]

\[
(1 - \tau^r)(w_t L_t + r_t K^p_t) - T_i - C^p_t - K^p_{t+1} + (1 - \delta^p)K^p_t = 0
\]

### 4.2 FIRMS

There is a large number of identical firms. The representative firm uses capital and labour services purchased from households to produce goods. We assume that the firm’s production function is constant returns to scale in private capital and labour and increasing returns to scale in total inputs. The production function takes a Cobb-Douglas form which is given by:

\[
Y_t = A_t (K^p_t)^{\theta} (H_t)^{1-\theta} (K^G_t)^{\nu}
\]

Where \( K^p_t \) private capital is public, \( K^G_t \) is the stock of public capital, \( H_t \) is number of hours worked, and \( A_t \) is an exogenous stochastic process for the state of technology at time \( t \). \( \theta < 1, \nu < 1 \) is the productivity of private and public capital respectively, and \((1-\theta)\) is the productivity of labour. When \( \nu = 0 \), the production function collapses to the standard case where the public capital is unproductive. When \( 0 < \nu < 1 \), the public capital becomes productive. The technology progress \( A_t \) is assumed to follow a AR(1) process:

\[
\log A_{t+1} = (1 - \rho_A) \log A_0 + \rho_A \log A_t + \varepsilon_{A,t+1}
\]

Where \( A_0 \) is a constant, \( \rho_A \) represents the persistence and \( \varepsilon_{A,t+1} \approx N(0, \sigma_A^2) \) is the normally distributed disturbances

\[
\Pi_t = Y_t - r_t K^p_t - w_t H_t
\]

It is assumed that firms act competitively in the goods market. Each representative firm earns gross income \( Y_t \), and pay interest \( r_t \), and wage \( w_t \), to capital and labour. The static optimisation problem for each representative firm is to choose \( \left\{ K^p_t, H_t \right\}_{t=0}^{\infty} \) in order to maximise profits subject to the technology constraint:
Max \{ Y_{t} - r_{t} K_{t}^{p} - w_{t} H_{t} \}

s.t. \[ Y_{t} - A_{t}(K_{t}^{p})^{\theta}(H_{t})^{1-\theta}(K_{t}^{G})^{\nu} \]

The FOC with respect to \( K_{t}^{p} \) is: \( r_{t} = \frac{\partial Y_{t}}{\partial K_{t}^{p}} \). This is the return to capital stock.

Similarly, the FOC with respect to \( H_{t} \) is: \( w_{t} = \frac{(1-\theta)Y_{t}}{H_{t}} \). This is the return to labour supply.

The profit of each firm is zero given the perfect competition in goods markets and the specification of the production function. This can be seen by substituting the returns to private capital and labour into the profit function.

\[ \Pi_{t} = Y_{t} - \frac{\partial Y_{t}}{\partial K_{t}^{p}} K_{t}^{p} - \frac{(1-\theta)Y_{t}}{H_{t}} H_{t} = 0 \]

### 4.3 GOVERNMENT

There is a central government which spends its income on public consumption and investment. Public consumption delivers positive utility to households. Public investment is used in augmenting public capital stock to serve for production. It is assumed that government expenditures are financed by means of taxes on incomes and lump sum taxes. The government’s budget constraint is then given by:

\[ G_{t} = C_{t}^{G} + I_{t}^{G} = \tau_{t} Y_{t} + T_{t} \]

The change in public capital stock follows the following process:

\[ K_{t+1}^{G} = I_{t}^{G} + (1-\delta^{G})K_{t}^{G} \]

We assume that government consumption and investment are subject to exogenous shocks that follow first order autoregressive processes:

\[ \log(C_{t+1}^{G}) = \rho_{C_{t}} \log(C_{t}^{G}) + \varepsilon_{C_{t}}^{G} \]

\[ \log(I_{t+1}^{G}) = \rho_{I_{t}} \log(I_{t}^{G}) + \varepsilon_{I_{t}}^{G} \]

### 4.4. AGGREGATE CONSISTENCY CONDITION

The aggregate condition of the economy is that the quantity of goods desired by households and the government is equal to the quantity of commodities supplied by firms. In our model setup, this implies that the aggregate consumption and investment of households and the government are equal to the aggregate production firms:

\[ C_{t}^{p} + I_{t}^{p} + C_{t}^{G} + I_{t}^{G} = Y \]

\[ K_{t+1}^{p} = Y_{t} - C_{t}^{G} - I_{t}^{G} - C_{t}^{p} - (1-\delta^{p})K_{t}^{p} \]
5. DECENTRALIZED COMPETITIVE EQUILIBRIUM

The decentralized equilibrium is defined as a sequence of market prices \( \{w_t, r_t\}_{t=0}^\infty \) and economic allocations \( \{Y_t, C_t, I_t, H_t, K_t^p, K_t^g\} \) where:

- All markets (goods, capital and labour) clear
- Households maximize their lifetime utilities, which gives optimality conditions
- Firms maximize their expected profits, which gives the production function and optimality conditions
- The government balances its budget, which solves \( \{T_t\}_{t=0}^\infty \) residually
- The evolution of private and public capital and the resources constraint are satisfied in each time.

5.1 SOLVING THE OPTIMIZATION PROBLEM

The typical optimizing household solves the following problem:

\[
U(C_t, H_t) = E \sum_{t=0}^\infty \beta^t \left[ \frac{\left( C_t^p + \omega C_t^g \right)^\gamma \left( 1 - H_t \right)^{-\alpha}}{1 - \delta} \right]
\]

Subject to a sequence of optimality conditions:

\[
Y_t = A_t H_t^{1-\delta} \left( K_t^p \right)^\gamma \left( K_t^g \right)^\gamma
\]

\[
K_{t+1}^p = (1 - \tau^Y) Y_t - C_t^p - (1 - \delta^p) K_t^p + T_t
\]

\[
K_{t+1}^g = (1 - \delta^g) K_t^g + I_t^g
\]

\[
\log(C_t^g) = \rho_{C^g} \log(C_t^g) + \epsilon_{C^g}
\]

\[
\log(I_t^g) = \rho_{I^g} \log(I_t^g) + \epsilon_{I^g}
\]

Hence the Bellman equation can be written as:

\[
V(K_t^p, K_t^g, G_t) = \max_{C_t^p, H_t, K_t^g} \left\{ U(C_t^p, H_t) + \beta E_t [V(K_{t+1}^p, K_{t+1}^g, G_{t+1})] \right\}
\]

The first order conditions (FOC):

\[
\frac{\partial U}{\partial C_t^p} + \frac{\partial K_{t+1}^p}{\partial C_t^p} \lambda_t = 0
\]
\[ H_i : \frac{\partial U}{\partial H_i} + \frac{\partial K_{i+1}^p}{\partial H_i} \lambda_i = 0 \]

The Envelopes conditions:

\[ K_i^p : \frac{\partial V(K_i^p, K_i^g, G_i)}{\partial K_i^p} - \frac{\partial K_{i+1}^p}{\partial K_i^p} \lambda_i = 0 \]

\[ K_i^g : \frac{\partial V(K_i^p, K_i^g, G_i)}{\partial K_i^g} - \frac{\partial K_{i+1}^g}{\partial K_i^g} \eta_i = 0 \]

The Transversality Conditions:

\[ \lim_{\mu \to \infty} E_{i+\mu} \left[ \beta^{1+\mu} E_{i+\mu} \left[ \frac{\partial V(t+1+\mu)}{\partial K_{i+1}^p} \right] K_{i+1+\mu}^p \right] = 0 \]

\[ \lim_{\mu \to \infty} E_{i} \left[ \beta^{1+\mu} E_{i+\mu} \left[ \frac{\partial V(t+1+\mu)}{\partial K_{i+1}^g} \right] K_{i+1+\mu}^g \right] = 0 \]

\[ \lambda_i = \beta^i E_{i} \left[ \frac{\partial V(K_{i+1}^p, K_{i+1}^g, G_{i+1})}{\partial K_{i+1}^p} \right] \]

\[ \eta_i = \beta^i E_{i} \left[ \frac{\partial V(K_{i+1}^p, K_{i+1}^g, G_{i+1})}{\partial K_{i+1}^g} \right] \]

The first-order condition with respect to \( C_i^p \) is:

\[ \frac{\alpha}{C_i^p + \alpha C_i^g} \left[ \left( C_i^p + \alpha C_i^g \right)^\alpha (1-h_i)^{1-\alpha} \right]^{1-\sigma} = \lambda_i \]

The first-order condition with respect to \( H_i \) is:

\[ \frac{\alpha - 1}{1 - H_i} \left[ \left( C_i^p + \alpha C_i^g \right)^\alpha (1-H_i)^{1-\alpha} \right]^{1-\sigma} = (1 - \tau^\gamma) \lambda_i w_i \]

\[ \lambda_i w_i = \frac{\alpha - 1}{1 - H_i} \left[ \left( C_i^p + \alpha C_i^g \right)^\alpha (1-H_i)^{1-\alpha} \right]^{1-\sigma} \]

The Envelopes conditions are:

\[ K_i^p : \beta^i E_i \left[ \lambda_{i+1} (1-\delta^p) + \lambda_{i+1} (1-\tau^\gamma) r_i \right] = \lambda_i \]

\[ \lambda_i = \beta E_i \lambda_{i+1} [(1-\delta^p) + (1-\tau^\gamma) r_i] \]

\[ K_i^g : \beta^i E_i \left[ \eta_{i+1} (1-\delta^g) \right] = \eta_i \]

\[ \eta_i = \beta E_i \eta_{i+1} [(1-\delta^g)] \]
The labor supply function of the household is:

\[
\frac{\partial u}{\partial H_t} = \frac{\alpha - 1}{1 - H_t} = \frac{(1 - \tau^y)\lambda_tw_t}{\lambda_t},
\]

\[
\frac{\partial u}{\partial C_t^p} = \frac{C_t^p + \alpha C_t^g}{1 - H_t} \left(\frac{\alpha - 1}{\alpha}\right) = (1 - \tau^y)w_t,
\]

\[
w_t = \frac{C_t^p + \alpha C_t^g}{1 - H_t} \left(\frac{\alpha - 1}{\alpha(1 - \tau^y)}\right).
\]

We have:

\[
\frac{\alpha}{C_t^p + \alpha C_t^g} \left[\left(C_t^p + \alpha C_t^g\right)^\sigma (1 - h_t)^{1 - \sigma}\right] = \lambda_t
\]

\[
\lambda_t = \beta E_t \lambda_{t+1} \left[(1 - \delta^p) + (1 - \tau^y)r_t\right]
\]

Substituting \(\lambda_t\) and \(\lambda_{t+1}\) in FOC relative to consumption:

\[
\frac{\alpha}{C_t^p + \alpha C_t^g} \left[\left(C_t^p + \alpha C_t^g\right)^\sigma (1 - h_t)^{1 - \sigma}\right] = \beta E_t \left[\frac{\alpha}{C_{t+1}^p + \alpha C_{t+1}^g} \left[\left(C_{t+1}^p + \alpha C_{t+1}^g\right)^\sigma (1 - h_{t+1})^{1 - \sigma}\right] \left[(1 - \delta^p) + (1 - \tau^y)r_t\right]\right]
\]

Which is the Euler equation for household consumption is:

\[
1 = \beta E_t \left[\frac{\partial U}{\partial C_{t+1}^p} \left[(1 - \delta^p) + (1 - \tau^y)r_t\right]\right]
\]

\[
E_t \left(\beta' \frac{\partial U}{\partial C_t^p}\right) = E_t \beta' \left[(1 - \delta^p)(1 - \tau^y)r_t\right]
\]

5.2 Steady-state of the Decentralized Competitive Equilibrium

\[
\frac{\bar{w}}{\bar{r}} = \frac{\bar{C}_t^p + \alpha \bar{C}_t^g}{1 - \bar{H}} \left(\frac{\alpha - 1}{\alpha(1 - \tau^y)}\right)
\]

\[
1 = \beta \left[(1 - \delta^p) + (1 - \tau^y)\bar{r}\right]
\]

\[
\bar{r} = \frac{\theta \bar{y}}{\bar{K}_t^p}
\]
\[ \bar{w} = \frac{(1-\theta)\bar{Y}}{\bar{H}} \]

\[ \delta^\pi = \frac{1}{\beta} - \frac{1}{\beta} \]

\[ \bar{I}^p = \delta^\pi \bar{k}^p; \delta^\pi = \frac{\bar{I}^p}{\bar{k}^p} \]

\[ \bar{I}^\pi = \delta^\pi \bar{k}^\pi; \delta^\pi = \frac{\bar{I}^\pi}{\bar{k}^\pi} \]

\[ \bar{Y} = \bar{A}\bar{H}^{1-\theta} \left( \bar{K}^p \right)^\theta \left( \bar{K}^\pi \right) \]

\[ \bar{Y} = \bar{C}^p + \bar{I}^p + \bar{C}^\pi + \bar{I}^\pi \]

\[ \bar{G} = \bar{C}^\pi + \bar{I}^\pi = \tau^y \bar{Y} + \bar{T} \]

Exogenous processes: \[ \bar{C}^\pi = \bar{C}^{\pi y} y \]

\[ \bar{I}^\pi = \bar{I}^{\pi y} y \]

The model will be solved to conduct a range of policy analysis. Following the RBC literature, we use the log-linearization method to obtain a numerical solution. This method takes two steps. The first is to approximate the non-linear system of the model by log-linearizing it around the steady state. Second, the resulting linear rational expectation model is solved using the method proposed by Klein [2000] (see appendices).

5.3 CALIBRATION

Following Kydland and Prescott [1982], our analysis will be based on calibration of the model to annual Tunisian data. Compared to industrialized economies which have standard parameter values, the calibration for the Tunisian economy faced a lack of relevant empirical estimates and limited quality of data. The calibrated parameter values are summarized in table 3 bellow.
Table 3: Model calibration

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Value</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.968</td>
<td>Hirata, Kim and Kose [2003], Ben Slimane [2007]</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Coefficient of relative risk aversion</td>
<td>2.61</td>
<td>Hirata, Kim and kose [2003], Ben slimane [2007]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Intertemporal elasticity of substitution in labour supply</td>
<td>3</td>
<td>Hirata, Kim and kose [2003], Greenwood et al [1988]</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Substitution parameter- public and private consumption</td>
<td>0.03</td>
<td>Authors estimation</td>
</tr>
<tr>
<td>$\delta^p$</td>
<td>Depreciation rate on private capital</td>
<td>0.05</td>
<td>Tunisian Institute of competitiveness and Quantitative studies</td>
</tr>
<tr>
<td>$\delta^G$</td>
<td>Depreciation rate on public capital</td>
<td>0.05</td>
<td>Tunisian Institute of competitiveness and Quantitative studies</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Productivity of private capital</td>
<td>0.6</td>
<td>Tunisian Institute of competitiveness and Quantitative studies</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Productivity of public capital</td>
<td>0.05</td>
<td>Tunisian Institute of competitiveness and Quantitative studies</td>
</tr>
<tr>
<td>$\tau^y$</td>
<td>Rate of income taxation</td>
<td>0.24</td>
<td>Tunisian Institute of competitiveness and Quantitative studies</td>
</tr>
<tr>
<td>$\rho^c$</td>
<td>Persistence of government consumption shock</td>
<td>0.985</td>
<td>Authors estimation</td>
</tr>
<tr>
<td>$\epsilon^c$</td>
<td>Standard deviation of public consumption shock</td>
<td>0.145</td>
<td>Authors estimation</td>
</tr>
<tr>
<td>$\rho^I$</td>
<td>Persistence of government investment shock</td>
<td>0.689</td>
<td>Authors estimation</td>
</tr>
<tr>
<td>$\epsilon^I$</td>
<td>Standard deviation of government investment shock</td>
<td>0.039</td>
<td>Authors estimation</td>
</tr>
</tbody>
</table>

Steady state ratios

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Description</th>
<th>Steady State Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{C}^p/Y$</td>
<td>Private consumption ratio to GDP</td>
<td>0.617</td>
<td>Authors calculation</td>
</tr>
<tr>
<td>$\bar{C}^G/Y$</td>
<td>Public consumption ratio to GDP</td>
<td>0.157</td>
<td>Authors calculation</td>
</tr>
<tr>
<td>$\bar{I}^p/Y$</td>
<td>Private investment ratio to GDP</td>
<td>0.126</td>
<td>Authors calculation</td>
</tr>
<tr>
<td>$\bar{I}^G/Y$</td>
<td>Public investment ratio to GDP</td>
<td>0.118</td>
<td>Authors calculation</td>
</tr>
<tr>
<td>$\bar{K}^p/Y$</td>
<td>Private capital ratio to GDP</td>
<td>0.942</td>
<td>Authors calculation</td>
</tr>
<tr>
<td>$\bar{K}^G/Y$</td>
<td>Public capital ratio to GDP</td>
<td>0.996</td>
<td>Authors calculation</td>
</tr>
</tbody>
</table>

5.4 MOMENT MATCHING

Before we analyse the model dynamics and policy implications, we first examine the fit of the model to the Tunisian data. That is, to check if this calibrated DSGE model can predict the main features of business cycles. We employ the standard assessment that has been used by the traditional RBC literature. We stimulate the model and compare the population moments of artificial data, i.e. standard deviation, relative deviation to output, first-order correlation and contemporaneous correlation with output, with those of actual data.
Table 4: Moments Of The Model And Data

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Private consumption</th>
<th>Private investment</th>
<th>Public consumption</th>
<th>Public investment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>3.06</td>
<td>1.93</td>
<td>11.45</td>
<td>12.20</td>
<td>12.50</td>
</tr>
<tr>
<td>model</td>
<td>4.58</td>
<td>2.78</td>
<td>10.35</td>
<td>13.56</td>
<td>11.04</td>
</tr>
<tr>
<td>Relative standard deviation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>-</td>
<td>0.63</td>
<td>3.74</td>
<td>3.89</td>
<td>4.08</td>
</tr>
<tr>
<td>model</td>
<td>-</td>
<td>0.60</td>
<td>2.25</td>
<td>2.96</td>
<td>2.40</td>
</tr>
<tr>
<td>Contemporaneous correlation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>1</td>
<td>0.79</td>
<td>0.76</td>
<td>0.30</td>
<td>0.72</td>
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<tr>
<td>model</td>
<td>1</td>
<td>1.03</td>
<td>0.66</td>
<td>0.59</td>
<td>0.85</td>
</tr>
<tr>
<td>First order autocorrelation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>0.51</td>
<td>0.34</td>
<td>0.60</td>
<td>0.46</td>
<td>0.61</td>
</tr>
<tr>
<td>model</td>
<td>0.77</td>
<td>0.66</td>
<td>0.12</td>
<td>0.62</td>
<td>0.59</td>
</tr>
</tbody>
</table>

By comparing the moments of the model and those of the data, we have the following findings. In general the population moments of the artificial data are quite consistent and similar with those of the actual data. First, the model generates output series that is as volatile as in actual Tunisian economy. Second, the standard deviations of spending components of output and private and public capital stock are also consistent with those in the data. The relative volatility moments also match the data quite well. In this context the model correctly predicts that private consumption is much less volatile than output while private investment is much more volatile than output. The persistence of the model generated data and the actual data do not match well. The first order autocorrelation of output, private consumption, and private investment show big differences.

The co-movement statistics are presented in table.3 under the contemporaneous correlation with output column. It is shown that the co-movement of most variables with output is well captured by the model. But the model mimics the co-movements between output and private consumption and public consumption less precisely.

6. MODEL DYNAMICS AND POLICY ANALYSIS

After assessing the fit of the model to data, we now study the dynamics of the model in response to the exogenous shocks. Our aim is to explain how exogenous fiscal policy shocks are amplified to generate aggregate fluctuations through the propagation mechanism of the model. The different scenarios are: a transitory increase in government purchases financed by lump-sum taxes, a transitory increase in government purchases financed by distortionary taxes and a permanent increase in public investment financed by lump-sum taxes.

6.1 A TEMPORARY INCREASE IN GOVERNMENT PURCHASES FINANCED BY LUMP SUM TAXES

The DSGE model enables us to examine the dynamic responses of variables to government spending shocks. In this experiment, the one period increase in government spending is financed by lump-sum taxes.

- During the period shock we found significant “crowding-out” effect and negative wealth effect. While the fiscal shock occurred, private consumption, leisure and investment are reduced because the increase in government consumption implies fewer
resources that are available in the economy for private agents. If the government spending is financed by lump-sum taxes, there is also a negative wealth effect due to the decrease of the disposal income of households. In both cases of “crowding-out” and negative wealth effect, private consumption is reduced and labour supply is raised.

- When the shock has ended, private investment is above the long-run equilibrium level because the economy needs to rebuild the capital stock. Private consumption and leisure are low along the entire transition path while labour increases more in the initial phase than does along the transition path. This finding is due to the fact that at the end of the shock period, taxes are reduced implying an increase of disposable income; thus consumption grows smoothly in line with the permanent income hypothesis, leisure increases and labour decreases.

Figure 1 shows the impulse responses to a standard deviation shock of government purchases. The one period increase in government consumption decreases private consumption and raises labour supply (the negative wealth effect dominates). The increase in labour supply decreases real wage and the negative wealth effect raises the interest rate. Given the short period jump in labour supply, output increases in the first several years and growth quickly decelerates and becomes negative.

Figure 1 Temporary increase in government purchases financed by lump-sum

![Graph showing impulse responses](image-url)
6.2 A TEMPORARY INCREASE IN GOVERNMENT PURCHASES FINANCED BY DISTORTIONARY TAXES

In this policy experiment, the government finances current expenditures from distortionary tax revenues. The increase in the tax rate reduces individual’s to work and invest, which results in a reduction of the tax base. As result, the tax rate must increase more than current expenditures to finance the additional public spending. There is a strong incentive to substitute intertemporally work effort and to reduce investment during this period.

Figure 2 reports the responses of the principal macroeconomic aggregates. Labour decreases during shock period, since wage is taxed more and hence is optimal to reschedule it. Then it smoothly goes back to steady state when the shock period is ended. Output exhibits the same dynamic as labour. Consumption decreases during the shock period because of income decreasing and recovery at the end of the transitory period. Wages and interest rate reflect the path of output. Investment declines during the shock period regarding the absorption of resources by government and because of low rates of return. The tax rate has to increase more than government expenditures and continues to grow as output declines to keep the budget balanced.

Figure 2: Temporary increase in government purchases financed by distortionary taxes
6.3 PERMANENT INCREASE IN PUBLIC INVESTMENT FINANCED BY LUMP-SUM TAXES

As shown in the production function, an increase in government capital affects the macroeconomic dynamics similarly to a productivity shock. Baxter and King distinguish between the short-run and long-run effects of public investment. In the long run the effect of public investment on output depends on: (i) the direct productivity effect or wealth effect due to the increase in productive public capital. Given the accumulation rule of public capital, this is subject to one year lag. (ii) A supply side effect due to the response of private capital and labour supply. In the short run three forces operate along the transition path: (i) the government absorbs resources, (ii) output increases as public capital increases, (iii) the marginal products of private capital and labour improve, leading to higher demand for capital and labour.

As seen in figure 3, consumption and leisure decrease due to the absorption of resources (negative wealth effect). In the long-run the additional supply of labour and capital stimulates output, and therefore consumption increases. Since leisure decreases, labour increases and the
marginal product of labour increases. Private investment permanently increases because its marginal product has increased. The return to investment is higher, hence more capital is accumulated. Output follows closely what happens to investment due to the increase in public capital. The real interest rate is higher but declining along the transition path. Initially it increased due to the increase in marginal product of capital. The wage decreases as a consequence of the increase in labour supply, and then slightly increases.

Figure 3: Permanent Increase In Public Investment Financed By Lump-Sum Taxes

![Graph showing consumption, private investment, and output over time.](image-url)
7. CONCLUSION

This paper has developed an RBC model to analyze the Tunisian macroeconomic fluctuations. The model has allowed us to understand business cycles in a dynamic stochastic general equilibrium framework. We extended the prototype RBC model by incorporating exogenous fiscal policy shocks and we calibrated it to annual Tunisian data for the period 1970-2009. The performance model was evaluated by implementing an assessment of moment matching. The effects of fiscal policy shocks on the dynamic of the model were identified by analyzing the impulse response functions.

Most of the work done on the effects of fiscal policy shocks either treats government spending as total government or studies one of these two components separately. Since one contribution of this paper is to take this composition seriously by establishing a distinction between the consumption and the investment expenditure components because these shocks were expected to generate very different macroeconomic effects. While both shocks lead to the
absorption of resources by the government, and thus generate a negative wealth effect, other factors come into play.

Our main findings are (i) In general the population moments of the artificial data are quite consistent and similar with those of the actual data, meaning that the model generates a reasonable overall account of the business cycle in the Tunisian economy. (ii) The economy’s response to an increase in government expenditure depends on how it is financed. The tax distortion has a strong negative effect compared to the lump sum taxes. Distortionary tax finance may lead to a decline in output, consumption, and investment. In contrast lump-same tax increase output and employment. (ii) For government investment, the effect differs from that of government consumption in that it increases output directly in the production function.

These findings lead us to conclude that the fiscal stimulus package through additional temporary government spending may help the Tunisian government to achieve both the social goals (respond to the short-term social demands) and stimulate economic activity and employment, particularly if spending takes the form of investment in infrastructure.

REFERENCES


APPENDICES

(a) ANALYTICAL SOLUTION OF STEADY STATE

\[\begin{align*}
\bar{A} &= \bar{A}_0 \\
\bar{C}^{gy} &= \bar{C}_0^{gy} \\
I^{gy} &= I_0^{gy} \\
\bar{r} &= \frac{1}{\beta} - (1 - \delta^p) \\
\end{align*}\]

\[
\bar{w} = \frac{\bar{C}^p + \alpha \bar{C}^g}{1 - \bar{H}} \left( \frac{\alpha - 1}{\alpha(1 - \tau^v)} \right) = \frac{(1 - \theta)\bar{Y}}{\bar{H}}
\]

\[
\bar{H} (\bar{C}^p + \alpha \bar{C}^g)(\alpha - 1) = (1 - \bar{H})\alpha(1 - \tau^v)(1 - \theta)\bar{Y}
\]

\[
\bar{H}((\bar{C}^p + \alpha \bar{C}^g)(\alpha - 1) + \alpha(1 - \tau^v)(1 - \theta)\bar{Y}) = \alpha(1 - \tau^v)(1 - \theta)\bar{Y}
\]

\[
\bar{H} = \frac{\alpha(1 - \tau^v)(1 - \theta)}{((\bar{C}^p + \alpha \bar{C}^g)(\alpha - 1) + \alpha(1 - \tau^v)(1 - \theta)\bar{Y})}
\]

Or \[\bar{C}^p = 1 - \bar{I}^p - \bar{C}^g - \bar{I}^g, \bar{C}^g = \bar{C}^{gy}, \bar{I}^g = \bar{I}^{gy}\]

\[
\frac{\bar{C}^p}{\bar{Y}} = \frac{\bar{C}^g}{\bar{Y}} = \frac{\bar{I}^p}{\bar{Y}} = \frac{\bar{I}^g}{\bar{Y}} = \frac{\delta^p \bar{K}^p}{\bar{Y}} = \frac{\delta^p \theta}{\bar{r}}
\]

Where \[\alpha(1 - \tau^v)(1 - \theta)
\]

\[
\bar{H} = \frac{\alpha(1 - \tau^v)(1 - \theta)}{\left(1 - \frac{\delta^p \theta}{\bar{r}} - (1 - \omega)\bar{C}^{gy} - \bar{I}^{gy}\right)\left((\alpha - 1) + \alpha(1 - \tau^v)(1 - \theta)\right)}
\]

\[
\bar{r} = \frac{\theta \bar{Y}}{\bar{K}^p} \rightarrow \bar{K}^p = \frac{\theta \bar{Y}}{\bar{r}}
\]

\[
\bar{I}^{gy} = \delta^g \bar{K}^g \rightarrow \bar{K}^g = \frac{\bar{I}^{gy} \bar{Y}}{\delta^g}
\]
Substituting $\bar{k}^p$ and $\bar{k}^s$ into the production function gives:

$$\bar{Y} = \bar{A} \bar{H}^{1-\theta} \left( \frac{\theta \bar{F}}{\bar{r}} \right)^{\theta} \left( \frac{\bar{I}^{gy} \bar{Y}^{v}}{\bar{\delta}^g} \right)^v$$

$$\bar{Y}^{1-\theta-v} = \bar{A} \bar{H}^{1-\theta} \left( \frac{\theta}{\bar{r}} \right)^{\theta} \left( \frac{\bar{I}^{gy}}{\bar{\delta}^g} \right)^v$$

$$\bar{Y} = \bar{A} \bar{H}^{1-\theta} \left( \frac{\theta}{\bar{r}} \right)^{\theta} \left( \frac{\bar{I}^{gy}}{\bar{\delta}^g} \right)^v \bar{Y}^{1-\theta-v}$$

Other variables can be derived as:

$$\bar{K}^p = \frac{\theta \bar{Y}}{\bar{r}}$$

$$\bar{I}^p = \delta^p \bar{K}^p$$

$$\bar{I}^p = \delta^p \left( \frac{\theta}{\bar{r}} \right)^{-\theta} \left( \frac{\bar{I}^{gy}}{\bar{\delta}^g} \right)^v$$

$$\bar{K}^g = \frac{\bar{I}^{gy} \bar{Y}}{\bar{\delta}^g}$$

$$\bar{I}^{gy} \bar{A} \left( \frac{\theta}{\bar{r}} \right)^{-\theta} \left( \frac{\bar{I}^{gy}}{\bar{\delta}^g} \right)^v$$

$$\bar{K}^g = \frac{\bar{I}^{gy}}{\bar{\delta}^g}$$
\[
\bar{w} = \frac{(1-\theta)\overline{y}}{H}
\]

\[
(1-\theta)A\left[\frac{\alpha(1-\tau^v)(1-\theta)}{\left(1-\frac{\delta^v \theta}{\overline{r}} - (1-\omega)\overline{c}^{gy} - \overline{I}^{gy}\right)(\alpha - 1) + \alpha(1-\tau^v)(1-\theta)}\right]^{1-\theta}\left[\frac{\theta}{\overline{r}}\left(\overline{I}^{gy}\right)^v\right]^{1-\theta-v}
\]

\[
\bar{w} = (1-\theta)A\left[\frac{\alpha(1-\tau^v)(1-\theta)}{\left(1-\frac{\delta^v \theta}{\overline{r}} - (1-\omega)\overline{c}^{gy} - \overline{I}^{gy}\right)(\alpha - 1) + \alpha(1-\tau^v)(1-\theta)}\right]^{-2(\theta + \theta^2 + \theta^v)}\left[\frac{\theta}{\overline{r}}\left(\overline{I}^{gy}\right)^v\right]^{1-\theta-v}
\]

\[
\overline{c}^s = \overline{c}^{gy}\overline{y}
\]

\[
\overline{y} = \overline{c}^{gy}A\left[\frac{\alpha(1-\tau^v)(1-\theta)}{\left(1-\frac{\delta^v \theta}{\overline{r}} - (1-\omega)\overline{c}^{gy} - \overline{I}^{gy}\right)(\alpha - 1) + \alpha(1-\tau^v)(1-\theta)}\right]^{-1-\theta}\left[\frac{\theta}{\overline{r}}\left(\overline{I}^{gy}\right)^v\right]^{1-\theta-v}
\]

\[
\overline{I}^s = \overline{I}^{gy}\overline{y}
\]

\[
\overline{y} = \overline{I}^{gy}A\left[\frac{\alpha(1-\tau^v)(1-\theta)}{\left(1-\frac{\delta^v \theta}{\overline{r}} - (1-\omega)\overline{c}^{gy} - \overline{I}^{gy}\right)(\alpha - 1) + \alpha(1-\tau^v)(1-\theta)}\right]^{-1-\theta}\left[\frac{\theta}{\overline{r}}\left(\overline{I}^{gy}\right)^v\right]^{1-\theta-v}
\]

\[
\overline{c}^{\rho} = \overline{y} - \overline{I}^{\rho} + \overline{c}^s + \overline{I}^s
\]

\[
\overline{I} = \tau^v\overline{y} - \overline{c}^s - \overline{I}^s
\]

(b) MODEL SOLUTION

Log-linearization

\[\ln(x_r) \approx \ln(x) + \frac{1}{x}(x_r - x)\] and \[\ln(x_r) - \ln(x) = \frac{(x_r - x)}{x}\]

\[F(x_r, y_r) = G(x_r, y_r)\]

\[F(\overline{x}, \overline{y}) + \overline{\frac{\partial F(\cdot)}{\partial x_r}}(x_r - \overline{x}) + \overline{\frac{\partial F(\cdot)}{\partial y_r}}(y_r - \overline{y}) \approx G(\overline{x}, \overline{y}) + \overline{\frac{\partial G(\cdot)}{\partial x_r}}(x_r - \overline{x}) + \overline{\frac{\partial G(\cdot)}{\partial y_r}}(y_r - \overline{y})\]
\[ \frac{\partial F(\cdot)}{\partial (x_i - \bar{x})} + \frac{\partial F(\cdot)}{\partial (y_i - \bar{y})} = \frac{\partial G(\cdot)}{\partial (x_i - \bar{x})} + \frac{\partial G(\cdot)}{\partial (y_i - \bar{y})} \]

\[ \frac{\partial F(\cdot)}{\partial \bar{x}} \frac{\partial (\cdot)}{\partial (x_i - \bar{x})} + \frac{\partial F(\cdot)}{\partial \bar{y}} \frac{\partial (\cdot)}{\partial (y_i - \bar{y})} = \frac{\partial G(\cdot)}{\partial \bar{x}} \frac{\partial (\cdot)}{\partial (x_i - \bar{x})} + \frac{\partial G(\cdot)}{\partial \bar{y}} \frac{\partial (\cdot)}{\partial (y_i - \bar{y})} \]

\[ \frac{\partial F(\cdot)}{\partial \bar{x}} \frac{\partial (\cdot)}{\partial (x_i - \bar{x})} + \frac{\partial F(\cdot)}{\partial \bar{y}} \frac{\partial (\cdot)}{\partial (y_i - \bar{y})} = \frac{\partial G(\cdot)}{\partial \bar{x}} \frac{\partial (\cdot)}{\partial (x_i - \bar{x})} + \frac{\partial G(\cdot)}{\partial \bar{y}} \frac{\partial (\cdot)}{\partial (y_i - \bar{y})} \]

Log-linearization of production function

\[ \frac{Y - \bar{Y}}{\bar{Y}} \approx \left( \frac{A - \bar{A}}{\bar{A}} \right) \bar{A} \bar{H} \theta \left( \bar{K}^p \right)^{1-\theta} \left( \bar{K}^g \right)^{\theta} \]

\[ + \theta \left( \frac{H - \bar{H}}{\bar{H}} \right) \bar{A} \bar{H} \theta - 1 \left( \bar{K}^p \right)^{1-\theta} \left( \bar{K}^g \right)^{\theta} - (1 - \theta) \frac{K^p - \bar{K}^p}{\bar{K}^p} \bar{A} \bar{H} \theta \left( \bar{K}^p \right)^{-\theta} \left( \bar{K}^g \right)^{\theta} \]

\[ + \phi \left( \frac{K^p - \bar{K}^p}{\bar{K}^p} \right) \bar{A} \bar{H} \theta \left( \bar{K}^p \right)^{1-\theta} \left( \bar{K}^g \right)^{\theta} \]

\[ \bar{y}_t = \tilde{a}_t + \tilde{\Theta}_t + (1 - \theta) \tilde{k}_t^p + v \tilde{k}_t^g \]

Log-linearization of first order condition relative to consumption

\[ \alpha(C_t^p + \omega C_t^g)^{(1-\sigma)-1}(1 - H_t)^{(1-\alpha)(1-\sigma)} = \lambda_t \]

\[ \alpha(\alpha((1-\sigma) - 1)(C_t^p + \omega C_t^g))^{(1-\sigma)-2}(1 - H_t)^{(1-\alpha)(1-\sigma)} \left( \frac{C_t - \bar{C}}{\bar{C}} \right) \bar{C} + \]

\[ \alpha \omega(\alpha((1-\sigma) - 1)(C_t^p + \omega C_t^g))^{(1-\sigma)-2}(1 - H_t)^{(1-\alpha)(1-\sigma)} \left( \frac{C_t^g - \bar{C}^g}{\bar{C}^g} \right) \bar{C}^g \]

\[ \alpha(1 - \alpha)(1 - \sigma) \left( \frac{C_t^p + \omega C_t^g}{\bar{C}^p + \omega \bar{C}^g} \right)^{(1-\sigma)-1}(1 - H_t)^{(1-\alpha)(1-\sigma)} \left( \frac{H_t - \bar{H}}{1 - \bar{H}} \right) \frac{\bar{H}}{1 - \bar{H}} = \lambda \left( \frac{\lambda_t - \bar{\lambda}}{\bar{\lambda}} \right) \]

\[ \alpha(C_t^p + \omega C_t^g)^{(1-\sigma)-1}(1 - H_t)^{(1-\alpha)(1-\sigma)} \left[ \alpha((1-\sigma) - 1) \bar{C}_t + \omega \alpha((1-\sigma) - 1) \bar{C}_t - (1 - \alpha)(1 - \sigma) \frac{\bar{H}}{1 - \bar{H}} \bar{H}_t \right] = \bar{\lambda} \lambda_t \]

\[ \xi_{ec} = \alpha((1-\sigma) - 1), \xi_{ec}^g = \omega \alpha((1-\sigma) - 1), \xi_{ec} = (1 - \alpha)(1 - \sigma) \]

\[ \xi_{ec}^p + \xi_{ec}^g \bar{C}_t - \xi_{ec} \frac{\bar{H}}{1 - \bar{H}} \bar{H}_t = \bar{\lambda}_t \]

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Log-linearization of first order condition relative to labor

\[(\alpha - 1)\left(C_r^p + \alpha C_r^g \right)^{(1-\sigma)}(1 - H) = \theta \lambda, H^\theta \left(K_r^p \right)^{-\theta} \left(K_r^g \right)^{\theta} \]

\[(\alpha - 1)(\alpha((1 - \sigma) - 1)\left(C_r^p + \alpha C_r^g \right)^{(1-\sigma)}(1 - H) = \theta \lambda, H^\theta \left(K_r^p \right)^{-\theta} \left(K_r^g \right)^{\theta} \]

\[(\alpha - 1)(\alpha((1 - \sigma) - 1)\left(C_r^p + \alpha C_r^g \right)^{(1-\sigma)}(1 - H) = \theta \lambda, H^\theta \left(K_r^p \right)^{-\theta} \left(K_r^g \right)^{\theta} \]

\[
(\alpha - 1)(1 - \sigma)\left(C_r^p + \alpha C_r^g \right)^{(1-\sigma)}(1 - H)^{(1-\sigma)} = \alpha \left(C_r^p + \alpha C_r^g \right)^{(1-\sigma)}(1 - \sigma)\left(C_r^p + \alpha C_r^g \right)^{(1-\sigma)}(1 - H)^{(1-\sigma)} = \theta \lambda, H^\theta \left(K_r^p \right)^{-\theta} \left(K_r^g \right)^{\theta} \]

\[
\zeta_{k_r} = \alpha((1 - \sigma) - 1), \zeta_{k_g} = \alpha \omega((1 - \sigma) - 1), \zeta_H = (1 - \alpha)(1 - \sigma), \psi = \theta, \psi_H = \theta(\theta - 1), \psi_{k_r} = \theta \lambda, \psi_{k_g} = \theta \lambda \]

\[
\zeta_{k_r} = \alpha((1 - \sigma) - 1), \zeta_{k_g} = \alpha \omega((1 - \sigma) - 1), \zeta_H = (1 - \alpha)(1 - \sigma), \psi = \theta, \psi_H = \theta(\theta - 1), \psi_{k_r} = \theta \lambda, \psi_{k_g} = \theta \lambda \]

Log-linearization of Euler equation

\[
\alpha(1 - \sigma)\left(C_r^p + \alpha C_r^g \right)^{(1-\sigma)} - (1 - \alpha)(1 - \sigma)H^\theta \left(K_r^p \right)^{-\theta} \left(K_r^g \right)^{\theta} = \beta E_0 \left[\alpha(1 - \sigma)\left(C_r^p + \alpha C_r^g \right)^{(1-\sigma)} - (1 - \alpha)(1 - \sigma)H^\theta \left(K_r^p \right)^{-\theta} \left(K_r^g \right)^{\theta} \right] \]

Log-linearization of labor supply

\[
W_t = \frac{C_r^p + \alpha C_r^g}{1 - H} \left(\frac{\alpha - 1}{\alpha(1 - \tau)} \right) \]

\[
- \hat{\psi}_t = \bar{C}_r^p + \bar{C}_r^g \left(\frac{1}{1 - H} \right)^{\hat{H}} \hat{H} \]

Log-linearization of Capital evolution

\[
\bar{K}_t^p = \frac{\bar{K}_t^p}{\hat{H}} (1 - \delta^p) \bar{k}_t^p \]

\[
\bar{K}_t^g = \frac{\bar{K}_t^g}{\hat{H}} (1 - \delta^g) \bar{k}_t^g \]

Log-linearization of Capital stock

\[
r_t = \frac{\theta Y}{K_t^p} \]

Labour supply
\[ w_t = \frac{(1-\theta)Y_t}{H_t} \]

\[ \tilde{w}_t = \tilde{y}_t - \tilde{h}_t \]

**Log-linearization the aggregate production firms**

\[ Y_t = C_t^p + I_t^p + C_t^G + I_t^G \]

\[ \tilde{Y}_t = \frac{\overline{C}^p}{\overline{Y}} \tilde{\epsilon}_t^p + \frac{\overline{I}^p}{\overline{Y}} \tilde{I}_t^p + \frac{\overline{C}^G}{\overline{Y}} \tilde{\epsilon}_t^G + \frac{\overline{I}^G}{\overline{Y}} \tilde{I}_t^G \]

\[ G_t = C_t^G + I_t^G = \tau^Y Y_t + T_t \]

\[ \tilde{g}_t = \frac{\overline{C}^g}{\overline{G}} \tilde{\epsilon}_t^G + \frac{\overline{I}^g}{\overline{G}} \tilde{I}_t^G = \tau^Y \frac{\overline{Y}^Y}{\overline{G}} \tilde{y}_t + \frac{\overline{T}}{\overline{G}} \tilde{l}_t \]

**Log-linearization Exogenous processes**

\[ E_t \tilde{a}_{t+1} = \rho \tilde{a}_t \]

\[ E_t \tilde{g}_{t+1} = \rho \tilde{g}_t \]

\[ E_t \tilde{c}_{t+1} = \rho \tilde{c}_t \]

**Where**

\[ \tilde{a}_t = \tilde{a}_t + \tilde{y}_t \]

\[ \tilde{c}_t = \tilde{c}_t + \tilde{y}_t \]

**Solve the linearized system**

Rewrite the linearized system in first-order form:

\[ -\tilde{w}_t = \tilde{c}_t + \tilde{c}_t + \frac{H}{1-H} \tilde{h}_t \]

\[ \alpha(\alpha-1)(1-\sigma)\tilde{c}_{t+1}^p + \alpha(\alpha-1)(1-\sigma)\omega \tilde{c}_{t+1}^g - (1-\omega)(1-\sigma)\frac{H}{1-H} \tilde{h}_{t+1} = \]

\[ = \beta E_0 \left[ \alpha(\alpha-1)(1-\sigma)\tilde{c}_{t+1}^p + \alpha(\alpha-1)(1-\sigma)\omega \tilde{c}_{t+1}^g - (1-\omega)(1-\sigma)\frac{H}{1-H} \tilde{h}_{t+1} \right] \]

\[ \tilde{I}_t^p = \frac{\overline{K}^p}{\overline{I}^p} \tilde{K}_{t+1}^p - \frac{\overline{K}^p}{\overline{I}^p} (1-\delta^p) \tilde{K}_t^p \]

\[ \tilde{I}_t^g = \frac{\overline{K}^g}{\overline{I}^g} \tilde{K}_{t+1}^g - \frac{\overline{K}^g}{\overline{I}^g} (1-\delta^g) \tilde{K}_t^g \]

\[ \tilde{Y}_t = \frac{\overline{C}^p}{\overline{Y}} \tilde{\epsilon}_t^p + \frac{\overline{I}^p}{\overline{Y}} \tilde{I}_t^p + \frac{\overline{C}^G}{\overline{Y}} \tilde{\epsilon}_t^G + \frac{\overline{I}^G}{\overline{Y}} \tilde{I}_t^G \]

\[ \tilde{g}_t = \frac{\overline{C}^g}{\overline{G}} \tilde{\epsilon}_t^G + \frac{\overline{I}^g}{\overline{G}} \tilde{I}_t^G = \tau^Y \frac{\overline{Y}^Y}{\overline{G}} \tilde{y}_t + \frac{\overline{T}}{\overline{G}} \tilde{l}_t \]
\begin{align*}
E_i \alpha_{t+1} &= \rho_i \tilde{\alpha}_i \\
E_i \tilde{\gamma}_i^{g^{y}} &= \rho_i \tilde{\gamma}_i^{g^{y}} \\
E_i \tilde{c}_i^{g^{y}} &= \rho_i \tilde{c}_i^{g^{y}} \\
\tilde{t}_i^{g} &= \tilde{t}_i^{g^{y}} + \tilde{y}_i \\
\tilde{c}_i^{g} &= \tilde{c}_i^{g^{y}} + \tilde{y}_i
\end{align*}

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